Nama : Mohammad Febryan Khamim

NRP : 5002221072

30/5/2024

Kuis Bu Nur Asiyah

Tugas Kuis 2

1). a). Tunjukkan $\overline{\nabla} \times (r^2 \overline{r}) = \overline{0}$

misal : $\overline{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$r.r = r^2 = x^2+y^2+z^2$

$r^2\overline{r} = (x^2+y^2+z^2).(x\hat{i}+y\hat{j}+z\hat{k})$

$= x(x^2+y^2+z^2)\hat{i} + y(x^2+y^2+z^2)\hat{j} + z(x^2+y^2+z^2)\hat{k}$

$$\overline{\nabla} \times (r^2\overline{r}) = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \times \left(x(x^2+y^2+z^2)\hat{i} + y(x^2+y^2+z^2)\hat{j} + z(x^2+y^2+z^2)\hat{k}\right)$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x(x^2+y^2+z^2) & y(x^2+y^2+z^2) & z(x^2+y^2+z^2) \end{vmatrix}$$

$= (2yz - 2yz)\hat{i} + (2xz - 2xz)\hat{j} + (2xy - 2xy)\hat{k}$

$= 0\hat{i} + 0\hat{j} + 0\hat{k}$

$= \overline{0}$ Jadi terbukti bahwa $\nabla \times (r^2\overline{r}) = 0$

b). Jika diketahui $\nabla\phi = ((2z^4 - 2xy)\hat{i} - x^2\hat{j} + 8xz^2\hat{k}$. Dapatkan persamaan bidang $\phi(x,y,z) = 0$.

$$\nabla\phi = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \cdot ((2z^4 - 2xy)\hat{i} - x^2\hat{j} + 8xz^2\hat{k})$$

※ $\frac{\partial}{\partial x}(\phi) = 2z^4 - 2xy$

$= \int 2z^4 - 2xy\, dx$

$= 2xz^4 - x^2y$

※ $\frac{\partial}{\partial y}(\phi) = -x^2$

$= -x^2$

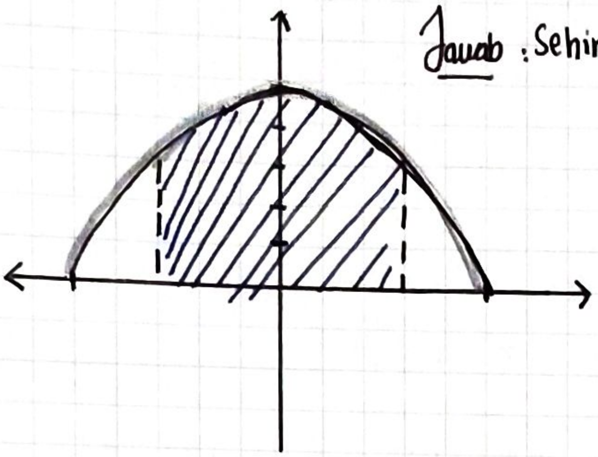
※ $\frac{\partial}{\partial z}(\phi) = 8xz^2$

$= \frac{8}{3}xz^3$

Sehingga, $\phi = 2xz^4 - x^2y - x^2 + \frac{8}{3}xz^3$

2). Medan vektor $F = (3 + 2x^2 y^{3/2})\hat{i} + (x^2\sqrt{y} - 3y^2)\hat{j}$.

Hitung $\int_c F$ di mana C adalah intasan kurva $y = \sqrt{25 - x^2}$, $-2 \leq x \leq 2$



Jawab : Sehingga, kurva C persamaan setengah lingkaran

$$\int_c F\, dr = \int_c (3 + 2x^2 y\sqrt{y})\, dx + (x^2\sqrt{y} - 3y^2)\, dy$$

$$= \int_{-3}^{3} (3 + 2x^2 y\sqrt{y})\, dx + 0$$

$$= \int_{-3}^{3} 3 + 16k^2\, dx$$

$$= 3x + \frac{16}{3}x^3 \Big|_{-3}^{3}$$

$$= 9 + 144 - (-9) - 144$$

$$= 288 + 18$$

$$= 306.$$

3). Dengan teorema Green, hitung usaha yang diperlukan untuk memindahkan partikel ... putaran : 1 make clips : $C = \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$ yang melewati medan gays.

Diketahui : $F = (2y - x^2)\hat{i} + (4x - 2y^2)\hat{j}$

$$\frac{\partial M}{\partial y} = 2 \qquad \frac{\partial N}{\partial x} = 4$$

$$W = \iint \frac{\partial N}{\partial x} - \frac{\partial m}{\partial y}\, dx\, dy$$

$$= \iint 4 - 2\, d$$

$$= \iint 2\, dx\, dy$$

$$= 2.4 \int_0^a \int_b^{\sqrt{1-(\frac{x}{a})^2}} dy\, dx$$

$$= 8 \int_0^a b\sqrt{1 - (\frac{x}{a})^2}\, dx$$

misal : $\frac{x}{a} = \cos\theta \rightarrow dx = -a\sin\theta\, d\theta$

BA $\rightarrow a = 0$; BB : $\frac{dx}{d\theta} = -a\cos\theta$

$$= 8\int_0^a -ab.\sin\theta \sin\theta\, du$$

$$= -8ab \int_0^{\pi/2} mle^2 Q$$

$$= -8ab\left(\frac{1}{4} - \frac{1}{4}\sin \pi\right)$$

Nama : Mohammad Febryan Khamim

NRP : 5002221072

30/5/2024

Pak Sentot

Tugas Kuis 2

1). Tunjukkan bahwa :

a). $\overline{\nabla} \times (r^2 \overline{r}) = \overline{0}$

misal : $\overline{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$r.r = x^2 + y^2 + z^2$

$r^2\,\overline{r} = (x^2+y^2+z^2).(x\hat{i} + y\hat{j} + z\hat{k})$

$= x(x^2+y^2+z^2)\hat{i} + y(x^2+y^2+z^2)\hat{j} + z(x^2+y^2+z^2)\hat{k}$

$\overline{\nabla} \times (r^2\overline{r}) = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \times \left(x(x^2+y^2+z^2)\hat{i} + y(x^2+y^2+z^2)\hat{j} + z(x^2+y^2+z^2)\hat{k}\right)$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x(x^2+y^2+z^2) & y(x^2+y^2+z^2) & z(x^2+y^2+z^2) \end{vmatrix}$$

$= (2yz - 2yz)\hat{i} + (2xz - 2xz)\hat{j} + (2xy - 2xy)\hat{k}$

$= 0\hat{i} + 0\hat{j} + 0\hat{k}$

$= \overline{0}$

Jadi, terbukti bahwa $\overline{\nabla} \times (r^2\overline{r}) = \overline{0}$

b). $\nabla . \overline{r} f(r) = 3f(r) + |r| \frac{df}{dr}$

↳ $\nabla\overline{r}f(r) = \nabla r\, f(r) + \overline{r}\, \Delta f(r)$

* $\nabla r = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right).(x\hat{i} + y\hat{j} + z\hat{k})$

$= 1 + 1 + 1$

$= 3$

$\nabla\overline{r}f(r) = \nabla r\, f(r) + r\, \Delta f(r)$

$= 3\, f(r) + (x\hat{i} + y\hat{j} + z\hat{k}) . \frac{(x\hat{i} + y\hat{j} + z\hat{k})}{(x^2+y^2+z^2)^{1/2}} \frac{df}{dr}$

$= 3\, f(r) + (x^2+y^2+z^2)^{1/2} \frac{df}{dr}$

$= 3\, f(r) + |r| \frac{df}{dr}$ [TERBUKTI]

2). Diberikan integral berikut ini : $\oint_C (2xy - x^2)dx + (x+y^2)dy$, jika C adalah kurva tertutup yang dibatasi oleh $y = x^2$, dan $a^3x = y^2$ (a adalah angka terakhir NRP).

a). Hitung integral tersebut dengan menggunakan Teorema Green!

$$\int_C = (2xy - x^2)dx + (x+y^2)\,dy$$

$$C = y = x^2 \quad \text{dan} \quad a^3x = y^2 \longrightarrow y^2 = 8$$

Titik potong :

$$8x = y^2$$
$$8x = x^4$$
$$x(x^3 - 8) = 0$$
$$x = 0 \vee x = 2$$

▶ Teorema Green

$$\oint_C Mdx + Ndy = \iint \left(\frac{\partial N}{\partial M_i} - \frac{\partial M}{dy}\right) dx\,dy$$

$$= \int_0^4 \int_{x^2}^{4^{3/2}\sqrt{x}} 1 - 2x \; dy\,dx$$

$$= \int_0^4 y - 2xy \Big|_{x^2}^{a^{3/2}\sqrt{x}} dx$$

$$= \int_0^a a^{3/2}\sqrt{x} - 2a^{3/2}x^{3/2} - x^2 - 2x^3 \, dx$$

$$= \frac{2}{3} a\sqrt{a}\; x\sqrt{x} - \frac{4}{5} a^{3/2} x^{5/2} - \frac{1}{3}x^3 + \frac{1}{2}x^4 \Big|_0^4$$

$$= \frac{2}{3} a^3 - \frac{4}{5} a^4 - \frac{1}{3} a^3 + \frac{1}{2} a^4$$

$$= \frac{1}{3} a^3 - \frac{3}{10} a^4$$

$$= \frac{1}{3} 4^2 - \frac{3}{10} 4^4$$

$$= \frac{832}{15} //$$

Jadi

$\oint_c (2xy-x^2)\,dx + (x+y^2)\,dy$

$= \int_{C_1} (2xy-x^2)dx + (x+y^2)dy + \int_{C_1} (2xy-x^2)dx + (x+y^2)dy$

$= \frac{1}{2}a^4 + \frac{1}{3}a^3 + \frac{1}{3}a^6 - \left(\frac{4}{5}a^4 + \frac{1}{3}a^6\right)$

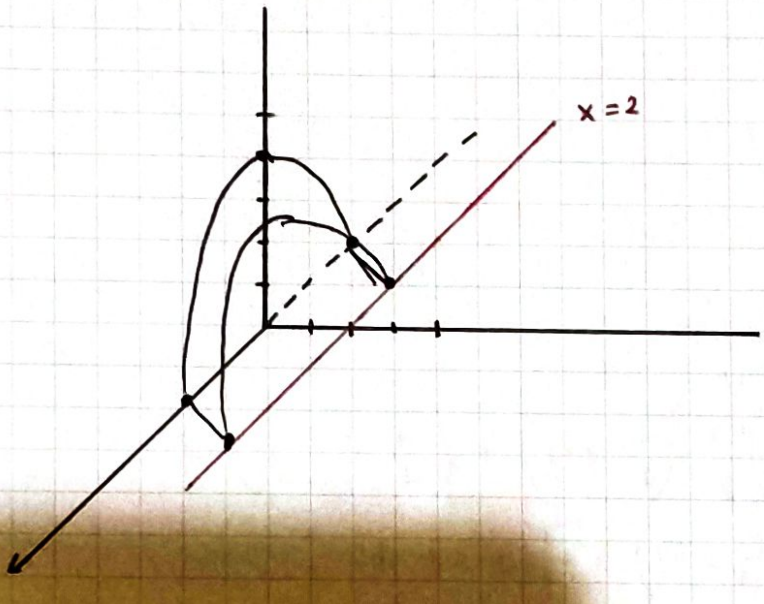
$= \frac{1}{3}a^3 - \frac{3}{10}a^4$

$= -\frac{832}{15}$ //

3). Diberikan vektor gaya $\bar{A} = (2xy+z)\hat{i} + y^2\hat{j} - (x+3y)\hat{k}$ dan S adalah permukaan daerah yang dibatasi oleh $z = a^2 - y^2$, $x=0$, dan $x=a$ dan bidang xy. (Dengan a = digit NRP terakhir)

a). Gambarkan Geometri batasan permukaan benda

$\Rightarrow z = 4 - y^2 \quad x = 0 \quad x = 2$



b). Hitung $\iint_S A\,\bar{n}\,dS$ dengan menggunakan teorema Gauss

$\bar{A} = (2xy+z)\hat{i} + y^2\hat{j} - (x+3y)\hat{k}$

div A $= 2y + 2y$

$= 4y$

$\iint_S \bar{A}\bar{n}\,ds = \iiint_V \text{div } A$

$= \int_0^2 \int_{-2}^{2} \int_0^{4-y^2} 4y\,dz\,dy\,dx$

$$= \int_0^2 \int_{-2}^2 4yz \Big|_0^{4-y^2} dy\, dx$$

$$= \int_0^2 \int_{-2}^2 4y(4-y^2)\, dy\, dx$$

$$= \int_0^2 \int_{-2}^2 16y - 4y^3\, dy\, dx$$

$$= 2\int_0^2 8y^2 - y^4 \Big|_0^2 dx$$

$$= 2\left[32 - 16\right]$$

$$= 2.16 \int_0^2 dx$$

$$= 2.16.2$$

$$= 64$$ //

c). Hitunglah $\iint_S A n\, dS$ langsung tanpa teorema Gauss!

$S_1 = z = a^2 - y^2$

$z_x = 0 \quad z_y = -2y$

$A = (2xy+z)\hat{i} + y^2\hat{j} - (x+3y)\hat{k}$

$P = 2xy + z \quad Q = y^2 \quad R = x + 3y$

$$\iint_{S_1} A n\, dS = \iint_D -P z_x - Q z_y + R\, dA$$

$$= \iint_D -(2xy+z).0 - (y^2)(-2y) + (-(x+3y))\, dA$$

$$= 2\int_0^4 \int_0^4 2y^3 - x - 3y\, dy\, dx$$

$$= 2\int_0^4 \frac{1}{2}y^4 - xy - \frac{3}{2}y^2 \Big|_0^4 dx$$

$$= 2\int_0^4 \frac{1}{2}(4)^4 - x(4) - \frac{3}{2}(4)^2\, dx$$

$$= 2\left[\frac{1}{2}(4)^4 x - \frac{1}{2}4x^2 - \frac{3}{2}(4)^2\right]\Big|_0^4$$

$$= 2\left[\frac{1}{2}.(4)^5 - \frac{1}{2}(4)^3 - \frac{3}{2}4^3\right]$$

$$= 4^5 - 4.4^3$$

$$= 4^5(4-1)$$

$$= 3.4^5$$ //

S_2 : $x = 2$ $\quad \bar{n} = \hat{i}$

$$\iint_S \bar{A}\bar{n}\, dS = \iint_{S_2} (2xy + z)\, dS$$

$$= 2\int_{y=0}^{2} \int_{z=0}^{4-y^2} 4y + z \; dz dy$$

$$= 2\int_0^2 4yz + \frac{1}{2}z^2 \Big|_0^{4-y^2} dy$$

$$= 2\int_0^2 4y(4-y^2) + \frac{1}{2}(4-y^2)^2\, dy$$

$$= 2\int_0^2 16y - 4y^2 + \frac{1}{2}\left[16 - 8y^2 + y^4\right] dy$$

$$= 2\int_0^2 16y - 4y^2 + 8 - 4y^2 + \frac{1}{2}y^4\, dy$$

$$= 2\left[8y^2 - \frac{4}{3}y^3 + 8y - \frac{4}{3}y^3 + \frac{1}{10}y^5 \Big|_0^2\right]$$

$$= 2\left[32 - \frac{32}{3} + 16 - \frac{32}{3} + \frac{32}{10}\right]$$

$$= 2\left[48 - \frac{64}{3} + \frac{32}{10}\right]$$

$$= 2\left[\frac{720 - 320 + 48}{15}\right]$$

$$= 2\left[\frac{448}{15}\right] \,//$$

S_3 : $x = 0$

$$\iint_{S_3} \bar{A}\bar{n}\, dS = \iint_{S_2} -(2xy + z)\, dz$$

$$= 2\int_0^2 \int_0^{4-y^2} -z\; dz dy$$

$$= 2\int_0^2 -\frac{1}{2}z^2 \Big|_0^{4-y^2} dy$$

$$= -\int_0^2 (4-y^2)^2\, dy$$

$$= -\int_0^4 2^4 - 2 \cdot 2^2 y^2 + y^4\, dy$$

$$= -\left(2^5 - \frac{2}{3}2^5 + \frac{1}{5}2^5\right)$$

$$= -\frac{18}{15} \cdot 2^5 \,//$$

No
Date

$S_4 : z = 0 \quad \bar{n} = -k$

$$\iint_{S_4} \bar{A}\,\bar{n}\,dS = \iint_{S_4} -(-(x+8y))\,dS$$

$$= 2\int_0^2 \int_0^2 x + 3y \; dy\,dx$$

$$= 2\int_0^4 xy + \frac{3}{2}y^2 \Big|_0^2 dx$$

$$= 2\int_0^4 2x + \frac{3}{2}.4 \; dx$$

$$= 2\int_0^4 2x + 6 \; dx$$

$$= 2\left(x^2 + \frac{3}{2}x^3\right)\Big|_0^2$$

$$= 2\left(\frac{4^2}{2} + \frac{3}{2}\,2^3\right)$$

$$= 4.\,2^3$$

$$= 32$$

Jadi, $\iint_S A\,n\,dS = 3.4^5 + .\,2\left[\frac{448}{15}\right] - \frac{18}{15}2^5 + 32$

$$= 3072 + 60 - 32 + 32$$

$$= 3672$$